

4.4

Graph $y = a(x - h)^2 + k$



The world's most important industry competition for fireworks manufacturers is L'International des Feux Loto-Québec. This event, also known as the Montréal Fireworks Festival, is held each summer in Montréal. The fireworks are synchronized to music that is also broadcast over a local radio station. Competing countries are judged on the synchronization, choice of music, and quality and originality of the visual display.

Paths of projectiles, such as rockets, balls, and fireworks, are often modelled using quadratic relations.

Investigate



Tools

- TI-83 Plus or TI-84 Plus graphing calculator
- grid paper

Technology Tip

Turn off all stat plots by pressing 2ND Y= for [STAT PLOT], selecting **4:PlotsOff**, and then pressing ENTER .

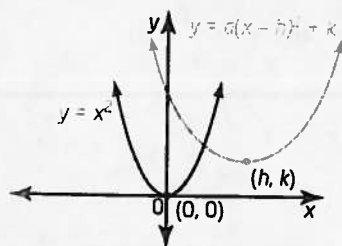
How do the graphs of $y = a(x - h)^2 + k$ and $y = x^2$ compare?

1. First, clear any graphed equations, and ensure all stat plots are turned off.
2. Use a standard window.
 - Press MODE and select **6:ZStandard**.
 - View the window settings by pressing WINDOW .
3. Graph the equations $y = x^2$, $y = 2x^2 - 5$, and $y = -x^2 + 2$.
4. a) Sketch all three graphs on the same set of axes.
 - b) Label the coordinates of the vertex and a second point on each parabola.
 - c) Describe the transformations.
 - d) Without using a graphing calculator, sketch the graph of $y = -2x^2 + 1$.
5. a) Clear all equations except $y = x^2$.
 - b) Graph the equation $y = (x - 2)^2 + 1$.

Plot1	Plot2	Plot3
$\text{Y}_1 = X^2$		
$\text{Y}_2 = 2X^2 - 5$		
$\text{Y}_3 = -X^2 + 2$		
$\text{Y}_4 =$		
$\text{Y}_5 =$		
$\text{Y}_6 =$		
$\text{Y}_7 =$		

6. a) Sketch the two graphs on the same set of axes.
 - b) Label the coordinates of the vertex and a second point on each parabola.
 - c) Draw the axis of symmetry for each parabola. Label each axis of symmetry with its equation.
 - d) Describe the transformations.
7. Without using a graphing calculator, sketch the graph of $y = (x - 1)^2 + 3$.
8. a) Repeat steps 5 and 6 for the equation $y = (x + 5)^2 - 4$.
 - b) Without using a graphing calculator, sketch the graph of $y = (x + 4)^2 + 2$.
9. a) Repeat steps 5 and 6 for the equation $y = 2(x - 1)^2 - 5$.
 - b) Without using a graphing calculator, sketch the graph of $y = -0.5(x + 2)^2 + 3$.
10. **Reflect** Write a summary of how to sketch a graph of a quadratic relation of the form $y = a(x - h)^2 + k$. Include a description of how to determine the coordinates of the vertex, the equation of the axis of symmetry, the values that x may take, and the values that y may take.

You can find the following from a quadratic relation of the form $y = a(x - h)^2 + k$:



- The vertex of the parabola is (h, k) , representing a horizontal translation of h units and a vertical translation of k units relative to the graph of $y = x^2$.
- The axis of symmetry of the parabola is the vertical line through the vertex with equation $x = h$.
- a indicates the vertical stretch or compression factor relative to the graph of $y = x^2$.
 - If $a > 0$, the parabola opens upward, and the vertex is the minimum point on the graph.
 - If $a < 0$, the parabola opens downward, and the vertex is the maximum point on the graph.

Example 1 Sketch the Graph of $y = a(x - h)^2 + k$

- a) Describe the properties of the parabola with equation $y = 2(x - 4)^2 - 3$.
- b) Sketch a graph of the parabola and label it fully.
- c) Describe the set of values that x may take.
- d) Describe the set of values that y may take.

Solution

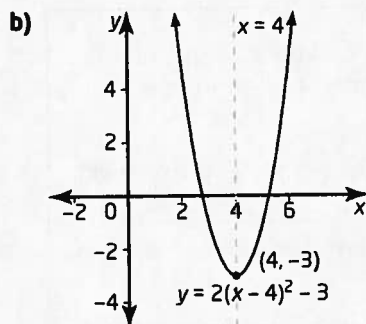
- a) Compare $y = 2(x - 4)^2 - 3$ with $y = a(x - h)^2 + k$.

Since $a = 2$, the graph of $y = 2(x - 4)^2 - 3$ will be stretched by a factor of 2 compared to the graph of $y = x^2$.

The parabola will open upward, since a is positive.

The vertex is (h, k) , or $(4, -3)$, and it is a minimum point.

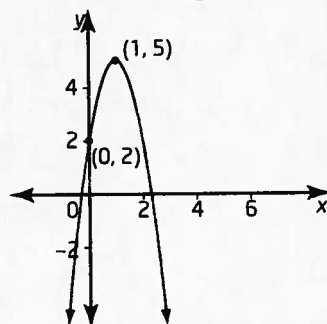
The equation of the axis of symmetry is $x = 4$.



- c) The graph shows that x may be any real number.
- d) The graph shows that y may be any real number greater than or equal to -3 , or $y \geq -3$.

Example 2 Write an Equation for a Graph

Determine an equation for the parabola shown.



Solution

The vertex is (1, 5), so $h = 1$ and $k = 5$.

The parabola opens downward, so a is negative.

Substitute the values for h and k into the equation $y = a(x - h)^2 + k$.

$$y = a(x - 1)^2 + 5$$

The parabola passes through the point (0, 2). Substitute $x = 0$ and

$y = 2$ and solve for a .

$$= a(0 - 1)^2 + 5$$

$$2 = a(-1)^2 + 5$$

$$2 = a + 5$$

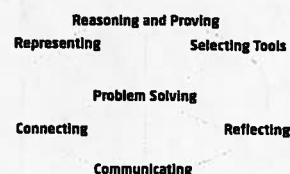
$$a = -3$$

An equation for the parabola is $y = -3(x - 1)^2 + 5$.

Example 3 Fireworks

At a fireworks display, a firework is launched from a height of 2 m above the ground and reaches a maximum height of 40 m at a horizontal distance of 10 m.

- Determine an equation to model the flight path of the firework.
- The firework continues to travel an additional 1 m horizontally, after it reaches its maximum height, before it explodes. What is its height when it explodes?
- At what other horizontal distance is the firework at the same height as in part b)?



Solution

- a) Sketch a graph of the situation.

The launch height of 2 m above the ground represents the point (0, 2).

The maximum height is 40 m at a horizontal distance of 10 m, so the vertex is (10, 40).

Substitute $h = 10$ and $k = 40$ into the equation

$$y = a(x - h)^2 + k.$$

$$y = a(x - 10)^2 + 40$$

Substitute $x = 0$ and $y = 2$ and solve for a .

$$2 = a(0 - 10)^2 + 40$$

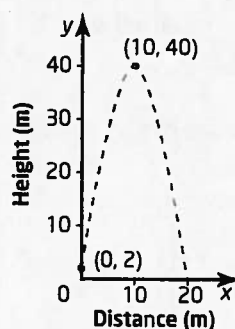
$$2 = a(-10)^2 + 40$$

$$2 = 100a + 40$$

$$-38 = 100a$$

$$a = -\frac{38}{100}$$

$$a = -0.38$$



An equation that models the flight path of the firework is $y = -0.38(x - 10)^2 + 40$, where x is the horizontal distance travelled, in metres, after the firework is launched and y is the height, in metres, above the ground.

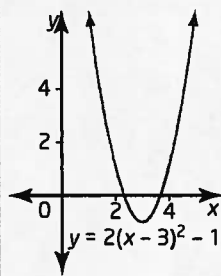
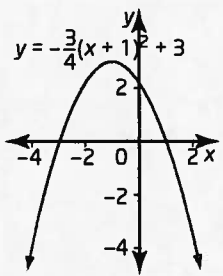
- b)** The firework travels an additional 1 m horizontally after its maximum height of 40 m at $x = 10$, so $x = 11$. Substitute $x = 11$ into $y = -0.38(x - 10)^2 + 40$.

$$\begin{aligned} y &= -0.38(11 - 10)^2 + 40 \\ &= -0.38(1)^2 + 40 \\ &= 39.62 \end{aligned}$$

The firework exploded at a height of 39.62 m.

- c)** Due to the symmetric property of a parabola, the firework is at the same height 1 m before the maximum point, or at a horizontal distance of 9 m.

Key Concepts

Property	$y = a(x - h)^2 + k$	$y = 2(x - 3)^2 - 1$	$y = -\frac{3}{4}(x + 1)^2 + 3$
Vertex	(h, k)	$(3, -1)$	$(-1, 3)$
Axis of symmetry	$x = h$	$x = 3$	$x = -1$
Stretch or compression factor relative to $y = x^2$	a	2	$-\frac{3}{4}$
Direction of opening	If $a > 0$, the parabola opens upward. The vertex is a minimum point. If $a < 0$, the parabola opens downward. The vertex is a maximum point.	Upward. $(3, -1)$ is a minimum point.	Downward. $(-1, 3)$ is a maximum point.
Graph	Parabola		
Values x may take	Any real number. Also depends on the situation.	Set of real numbers.	Set of real numbers.
Values y may take	If $a > 0$, then $y \geq k$. If $a < 0$, then $y \leq k$. Also depends on the situation.	$y \geq -1$	$y \leq 3$

